MAT1322

Lecture 8

Differential Equations part 2

30/01/2017

GENERAL FUNCTION: $\underset{\frac{dy}{dx}}{y'} = F(\underset{\text{independent variable}}{x}, y)$ ← function in x

Ex $y' = 5x$

$\frac{dy}{dx} = 5x$

$dy = 5x\,dx$ ← take integral of both sides

$\int dy = \int 5x\,dx$

$y = \frac{5x^2}{2} + C$ ✱ don't forget +C!

Task: determine function y

Method: separating variables

→ get all x on one side, all y on other side

INITIAL VALUE PROBLEMS

Ex $y\,dy = \frac{dx}{x}$, $y(1) = 2$

$\int y\,dy = \int \frac{1}{x}\,dx$

$\cancel{\frac{y^2}{2} = \ln|x| + C}$

$y^2 = 2\ln|x| + 2C$

$y = \pm\sqrt{2\ln|x| + 2C}$ ← "family" of solutions

✱ input initial value ✱

$2 = \pm\sqrt{2\ln(1) + 2C}$

$4 = 2C$ ✱ take positive part of root (don't carry ±)

$C = 2$

So, $y = \pm\sqrt{2\ln|x| + 4}$ is the specific solution

DIRECTION FIELDS (9.2)

Ex $\frac{dy}{dt} = y(t) + t^2$

$y' = y + t^2$

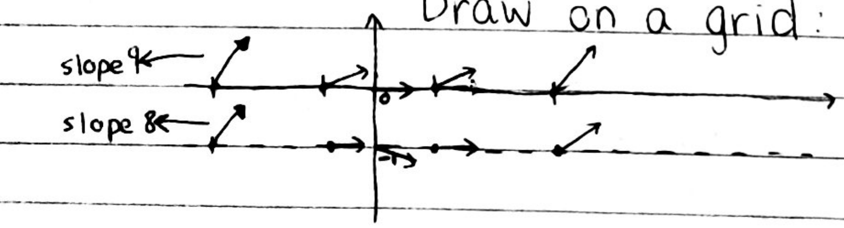
↓ slope of y at t

Create a table of values

y(t) \ t	−3	−1	0	1	3
0	9	1	0	1	9
−1	8	0	−1	0	8

← these #s will be given on an exam

Draw on a grid:



✱ at any point, there is a function ~~with a solution~~ going through with a specific solution

A direction field allows us to describe the behaviour of a solution without determining the function

Ex 2 $y' = 3xy$

$$\frac{dy}{dx} = 3xy$$

$$\int \frac{dy}{y} = \int 3x\,dx$$

$$\ln|y| = \frac{3}{2}x^2 + C$$ * solve for y

$$|y| = e^{\frac{3}{2}x^2 + C}$$

$$y = \pm e^{\frac{3}{2}x^2 + C}$$

$$y = \pm e^{\frac{3}{2}x^2} \cdot e^{C}$$ set $k = \underbrace{e^{C}}_{>0 \text{ for all } C}$

$$y = ke^{\frac{3}{2}x^2}$$

EULER'S METHOD

If we want to estimate the value at a specific point, we can use Euler's method. (only for initial value problems)

$$y_{n+1} = y_n + hF(x_n, y_n)$$

where h is the step size and F is the slope at (x_n, y_n), which are generally given as an initial value.

Ex
- $y' = xy - x^2$
- step size: $0.2 = h$
- initial value: $y(0) = 1$

Calculate $y(0.4)$

$x_0 = 0, \quad y_0 = 1$

$y_{n+1} = y_n + h \cdot F(x_n, y_n)$

$y_1(0) = 1 + 0.2\,(0 \cdot 1 - 0^2)$

$y_1(0) = 1$

increasing step size

$y_2(0 + 0.2) = 1 + 0.2\,F(0.2 \cdot 1 - 0.2^2)$

$\doteq 1.032$

$\therefore y(0.4) = 1.06528$

$y_3(0.2 + 0.2) = 1.032 + 0.2 \cdot F(0.2, 1.032)$

$y_3(0.4) = 1.06528$